Physical Optics

Object: To study the intensity patterns formed by single and double slits and by diffraction gratings.

Apparatus: Diode laser, single slit, double slit, diffraction grating, aperture, light sensor, rotary motion sensor, 1.2 m optical bench, screen, PASCO Interface, computer, and Data Studio Software.

FOREWORD

Consider two identical point sources of wave disturbances S_1 and S_2 that are vibrating in phase. If the source of one of these wave disturbances is a fixed distance and the other a variable distance from a distant screen where they combine, the result is summarized in **Figure 1**. The two wave disturbances are shifted slightly rather than being superimposed for the sake of clarity.

The point to be made by Figure 1 is that as the path difference between the two sources assumes different values, the resultant disturbance where the two waves combine at the screen goes from total reinforcement to total cancellation back to total reinforcement in a continuous manner and then repeats this cycle.

If both of the sources of wave disturbances are fixed relative to the screen and the path difference is achieved by having the wave disturbances combine at various distant points P on the screen, the result is summarized in **Figure 2**.

Once again the point to be made from Figure 2 is that as P is moved away from $y = 0$ (the direction doesn't matter) the difference in path length of the two wave disturbances and hence the phase difference between the two waves assumes different values. Consequently the net result at the screen goes from total reinforcement (constructive interference) to total cancellation (destructive interference).

In Figure 2, the light sources are a finite distance from the screen, hence the paths of wave disturbances are not parallel to each other. This case is called Fresnel Diffraction. A simplification occurs if the sources and the screen are a large enough distance from the diffracting aperture that it may be assumed that the light enters and leaves this aperture in a parallel manner. This limiting case is called **Fraunhofer Diffraction**. For the remainder of the experiment we shall use only Fraunhofer Diffraction. **Figure 3** shows the resultant intensity pattern for Fraunhofer Diffraction of a single slit. In this figure the slit of width a is divided into N parallel strips of width ΔX. Each strip acts as a source of wavelets (similar to S_1 and S_2) and produces a characteristic disturbance at P (located by a particular θ).

The minima occur when all of the light emerging from the slices (considered as individual sources) with a particular angle θ interferes in a totally destructive manner at the screen. **Figure 4** makes this fact a little more explicit. The solid lines show how the first minimum occurs and the dashed lines show how the second minimum occurs.

The light emerging from the first slice of the top and bottom half of the slit will cancel when the path difference is $\lambda/2$. If light from the first slice of the top and bottom of the slit have a path difference of $\lambda/2$ and hence cancel, then the light emerging from the second slice of the top and bottom of the slit parallel to this light have a path difference of $\lambda/2$ and hence cancels, etc. for the third and fourth slices of the top and bottom half of the slit. The physical situation may be expressed mathematically as

(1) $$path\ difference = \frac{a}{2}\,Sin\boldsymbol{\theta} = \frac{\lambda}{2} \qquad \textbf{first minimum}$$

or

(2) $$a\,Sin\boldsymbol{\theta}\ =\ \lambda \qquad \textbf{first minimum}$$
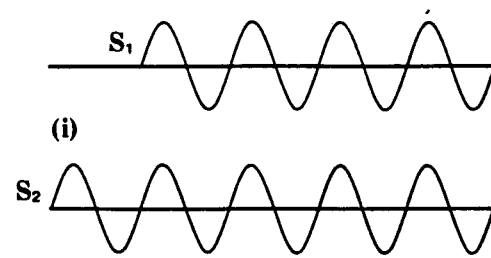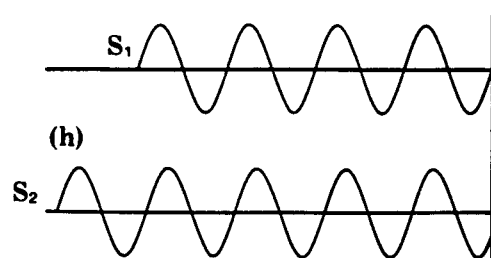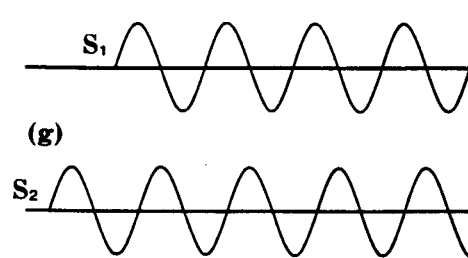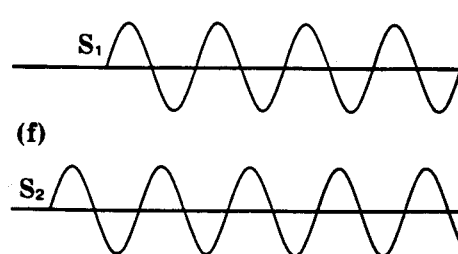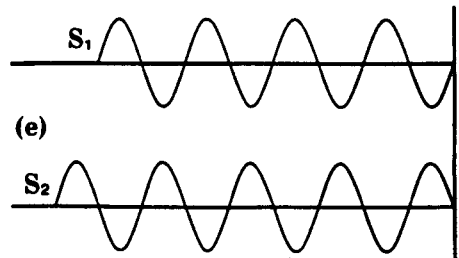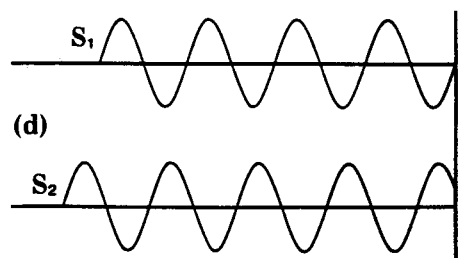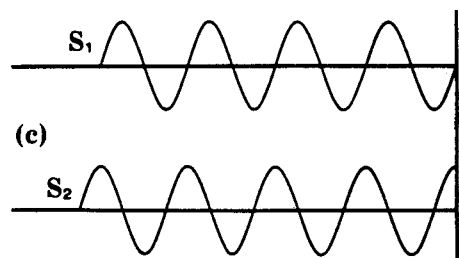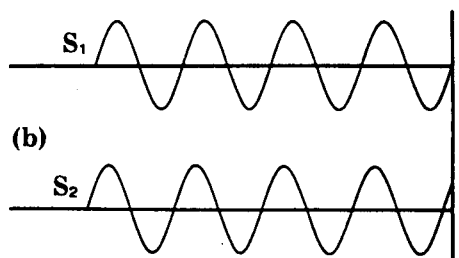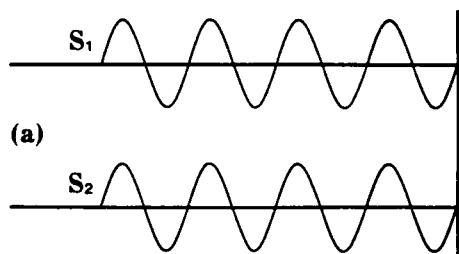
Drawing	Path Difference	Interference
a	0	Total reinforcement
b	$\lambda/8$	Partial cancellation
c	$\lambda/4$	Partial cancellation
d	$3\lambda/8$	Partial cancellation
e	$\lambda/2$	Total cancellation
f	$5\lambda/8$	Partial cancellation
g	$3\lambda/4$	Partial cancellation
h	$7\lambda/8$	Partial cancellation
i	λ	Total reinforcement

Figure 1

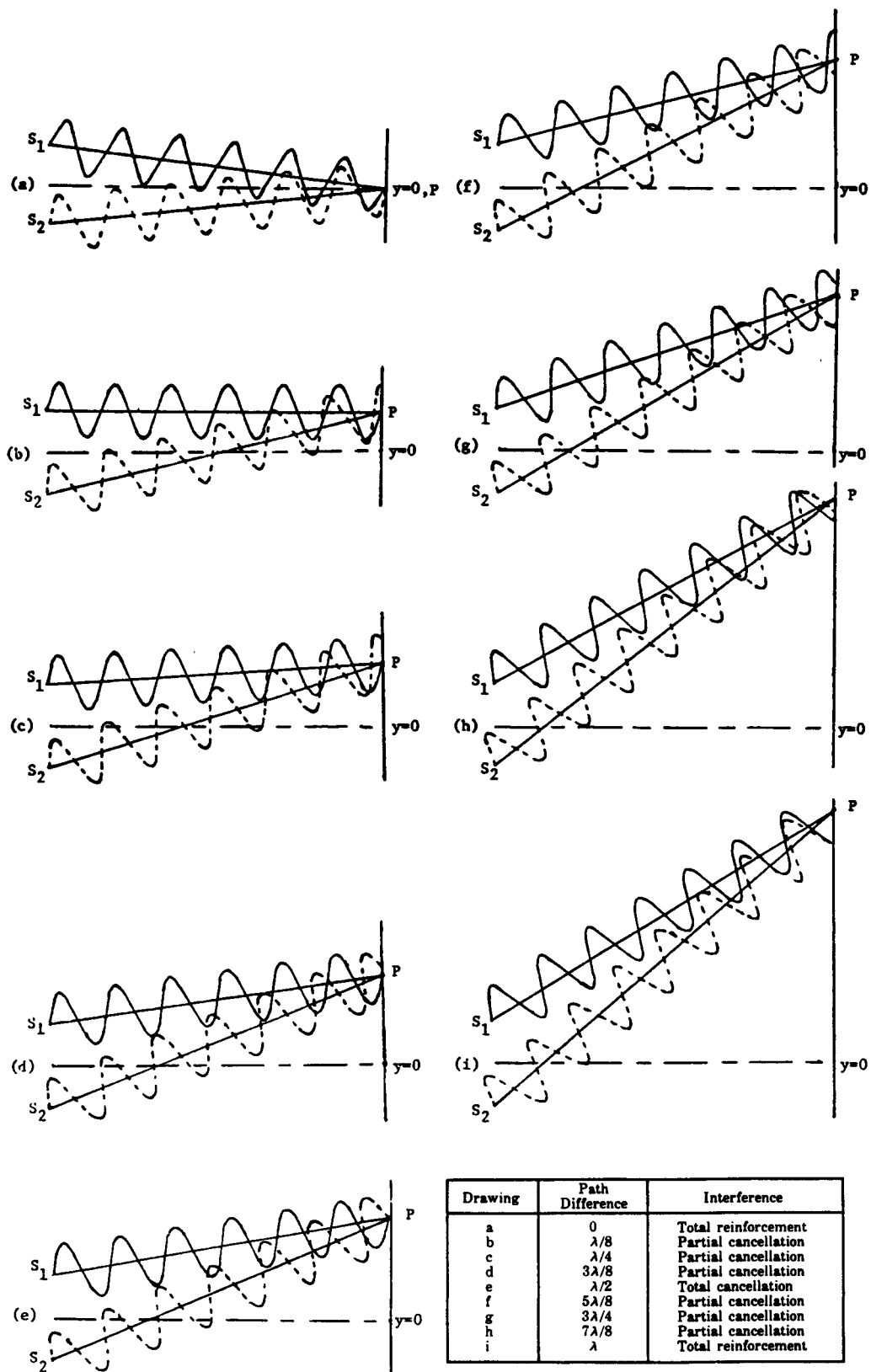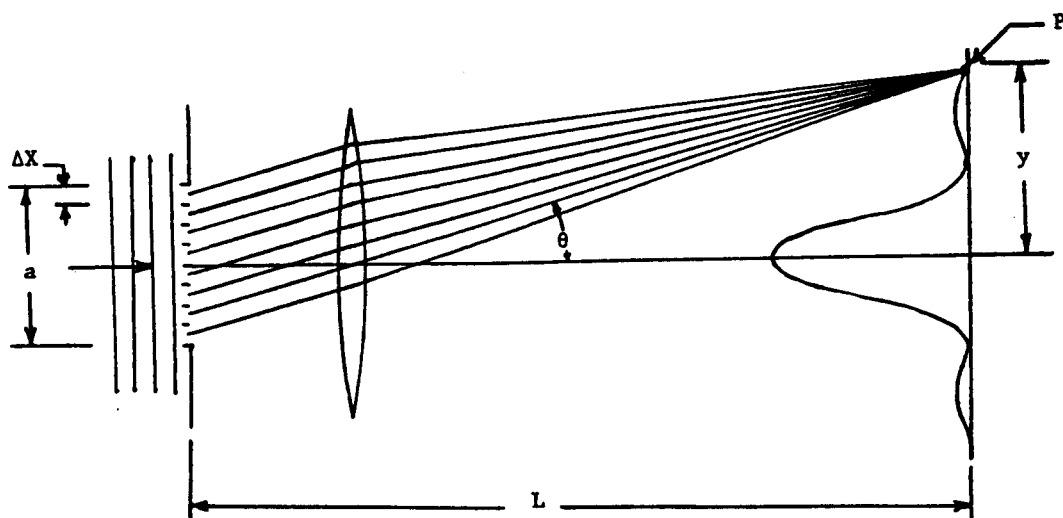| Drawing	Path Difference	Interference
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e	$\lambda/2$	Total cancellation
f	$5\lambda/8$	Partial cancellation
g	$3\lambda/4$	Partial cancellation
h	$7\lambda/8$	Partial cancellation
i	λ	Total reinforcement

Figure 2

Figure 3

The second minima may be accounted for noting that the light from the first slice of the first and second fourths of the slit will cancel when the path difference is $\lambda/2$. If light from the first slice of the first and second fourths of the slit have a path difference of $\lambda/2$ and hence cancel, then the light emerging from the second slice of the first and second fourths of the slit parallel to this light have a path difference of $\lambda/2$ and hence cancel. This process continues for the top half of the slit, i.e., the light from a particular slice in the first and second fourths being cancelled due to the fact that their path difference is $\lambda/2$. The process repeats itself for the bottom half of the slit, i.e., for the third and fourth fourths of the slit. The physical situation may be expressed mathematically as

(3) $$\text{path difference} = \frac{a}{4}\,\text{Sin}\theta = \frac{\lambda}{2},$$ **second minimum**

   or

(4) $$a\,\text{Sin}\theta = 2\lambda$$ **second minimum**

This process may be continued, however, looking at equations **(2)** and **(4)** it appears that enough work has been done to establish a pattern, namely

(5) $$a\,\text{Sin}\theta = n\lambda$$ $n = 1,2,3....$ **minima**

From the geometry of the physical situation one may write

(6) $$\text{Sin}\theta \cong \text{Tan}\theta = \frac{y}{L}.$$ $n = 1,2,3....$ **minima**

Combining equations **(5)** and **(6)**, we obtain

(7) $$y = \frac{n\lambda L}{a}.$$ $n = 1,2,3,...$ **minima**

The results for various values of n are summarized in the following chart.

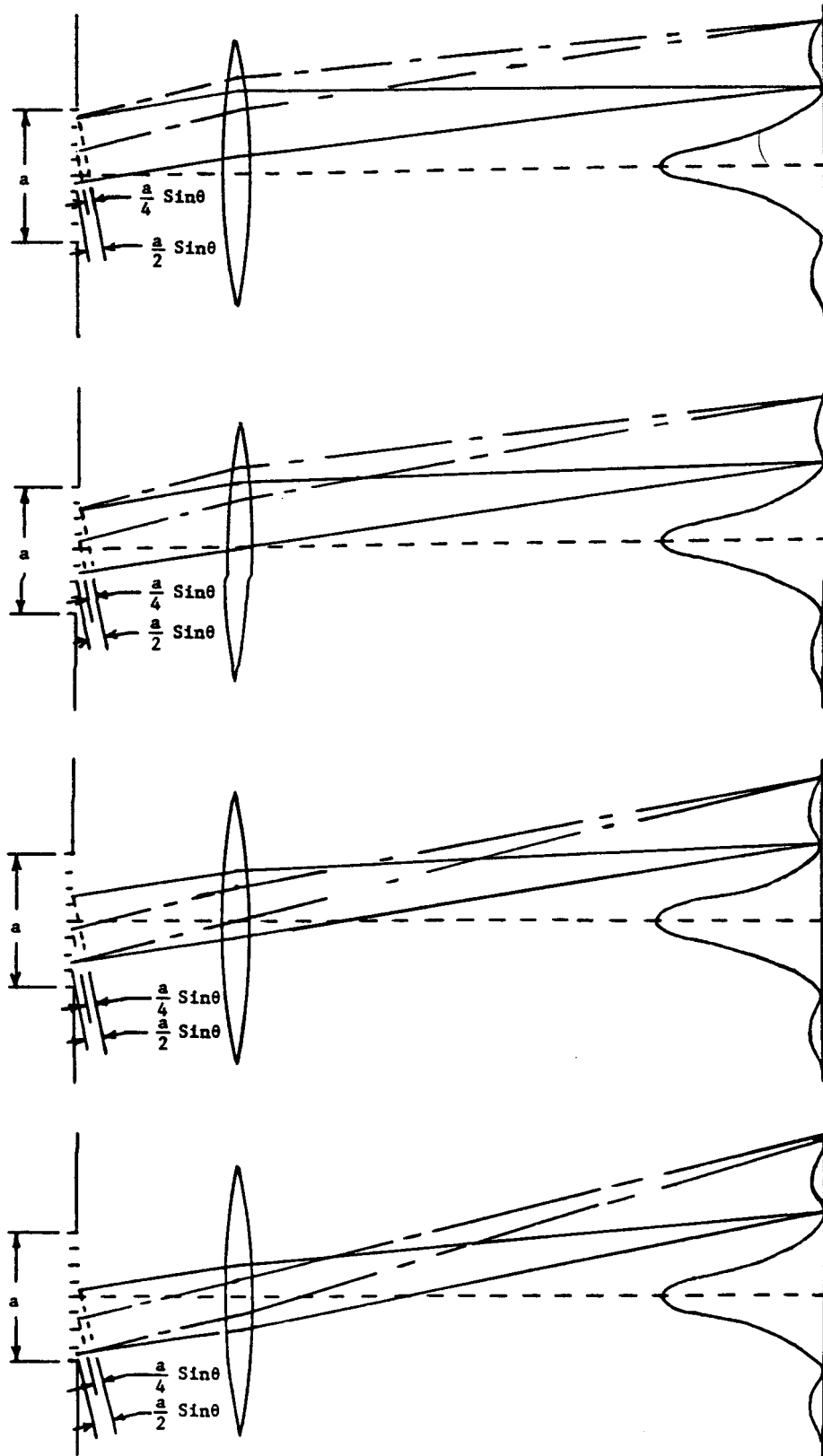| n	y	Location of P	Interference
1	$\lambda L/a$	First Minimum	Total Cancellation
2	$2\lambda L/a$	Second Minimum	Total Cancellation
3	$3\lambda L/a$	Third Minimum	Total Cancellation
.	.	.	.
.	.	.	.
.	.	.	.

3L–4

Figure 4

If we now open another slit (i.e., have a double slit arrangement) near the first single slit, the physical situation and the resultant intensity on the screen is as shown in **Figure 5**. Note that the diffraction pattern of the first single slit is still present, however, light from the second source interferes with the light from the first slit to form the interference fringes inside the diffraction envelope. P is an arbitrary point on the screen, a distance r_1 and r_2 from the slits S_1 and S_2, respectively. If the two slits are considered to be similar to the two point sources in Figures 1 and 2, then we appreciate the following statement. As θ assumes increasingly larger values (i.e., for different positions P or for different values of y) the distance $S_2 b$ will alternately be an integral number of half-wavelengths and whole wavelengths, hence alternately causing destructive and constructive interference respectively on the screen.

Constructive interference for the double slit occurs when the path difference is an integral number of whole wavelengths. Looking at Figure 5, we see that the path difference is

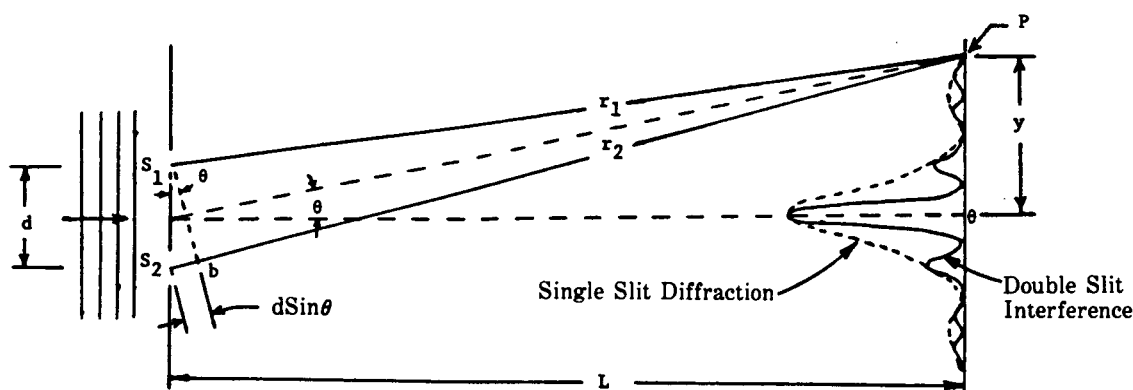**(8)** $$S_2\, b = d\, \mathrm{Sin}\theta .$$

Figure 5

According to the observations made from Figures 1 and 2, when this path difference is an integral number of whole wavelengths, the resultant combination on the screen is one of total reinforcement (constructive interference). This is expressed mathematically by

(9) $$d\, \mathrm{Sin}\theta = m\lambda \qquad\qquad m = 0,1,2,... \text{ maxima}$$

Inserting the geometry of the physical situation as expressed by equation **(6)** into equation **(9)** obtain

(10) $$y = \frac{mL\lambda}{d} . \qquad\qquad m = 0,1,2,... \text{ maxima}$$

The results for various values of m are summarized in the following chart.

m	Path Difference	y	Location of P	Interference
0	0	0	Central max.	Complete reinforcement
1	λ	$\lambda L/d$	First secondary max.	Complete reinforcement
2	2λ	$2\lambda L/d$	Second secondary max.	Complete reinforcement
3	3λ	$3\lambda L/d$	Third secondary max.	Complete reinforcement
•	•	•	•	•
•	•	•	•	•
•	•	•	•	•

3L–6

Equation (**10**) may also be used to obtain an expression for the double-slit fringe separation by letting m accept two successive values and then subtracting the results to get

(**11**)
$$\Delta y = \frac{L\lambda}{d}.$$

According to Equation (**10**), the central interference fringe maximum occurs when

(**12**)
$$y = 0,$$

the first secondary interference fringe maximum occurs when

(**13**)
$$y = \frac{L\lambda}{d}.$$

and the second secondary interference fringe maximum occurs when

(**14**)
$$y = \frac{2L\lambda}{d}.$$

If we assume that the interference fringe minima are located midway between these maxima then the first and second interference fringe minima occur at

(**15**)
$$y = \frac{L\lambda}{2d},$$

and

(**16**)
$$y = \frac{3L\lambda}{2d},$$

respectively. Or, in general, the interference fringe minima are located by

(**17**)
$$y = \left(m - \frac{1}{2} \right) L\lambda / d .$$
$$\qquad m = 1,2,3,... \text{ minima}$$

We would now like to determine the number of interference fringes under the central maximum of the single slit diffraction pattern. From Figure 4 we see that the y value for the first single slit pattern minimum ($n = 1$) must coincide with the y value for some number of interference fringe minima. Thus, we equate Eq (**7**) for the case of $n = 1$ and Eq (**17**) to obtain

(**18**)
$$\frac{\lambda L}{a} = \left(m - \frac{1}{2} \right) \frac{\lambda L}{d},$$

or

(**19**)
$$m = \frac{d}{a} + \frac{1}{2}.$$

If $m = 1, 2, 3, ...$, this means that the first, second, third interference fringe minimum coincided with the single slit minimum and there are one, three, five interference fringes, respectively, under the central maximum of the single slit pattern.

A logical extension of this two slit work is to increase the number of slits from two to a larger number N. Such an arrangement is called a diffraction grating. Analogous to the two slit case, the intensity pattern that results when monochromatic light of wavelength λ falls on a grating consists of a series of interference fringes. Since the diffraction grating can be thought of as an array of N slits, the mathematics for the N slices of the single slit should be applicable, hence, maxima will occur when the path difference ($d \sin\theta$) between rays from adjacent slits is given by

(**20**)
$$d \sin\theta = n\lambda$$
$$\qquad n = 0,1,2,... \text{ maxima}$$

If there are N slits per centimeter on the grating, then the distance d between slits is

(**21**)
$$d = \frac{1}{N},$$

and we get

(22) $$\text{Sin}\theta = n\lambda N \, .$$

The geometry for the diffraction grating is shown in **Figure 6** and is such that the approximation $\text{Sin}\theta \cong \tan\theta$ cannot be made.
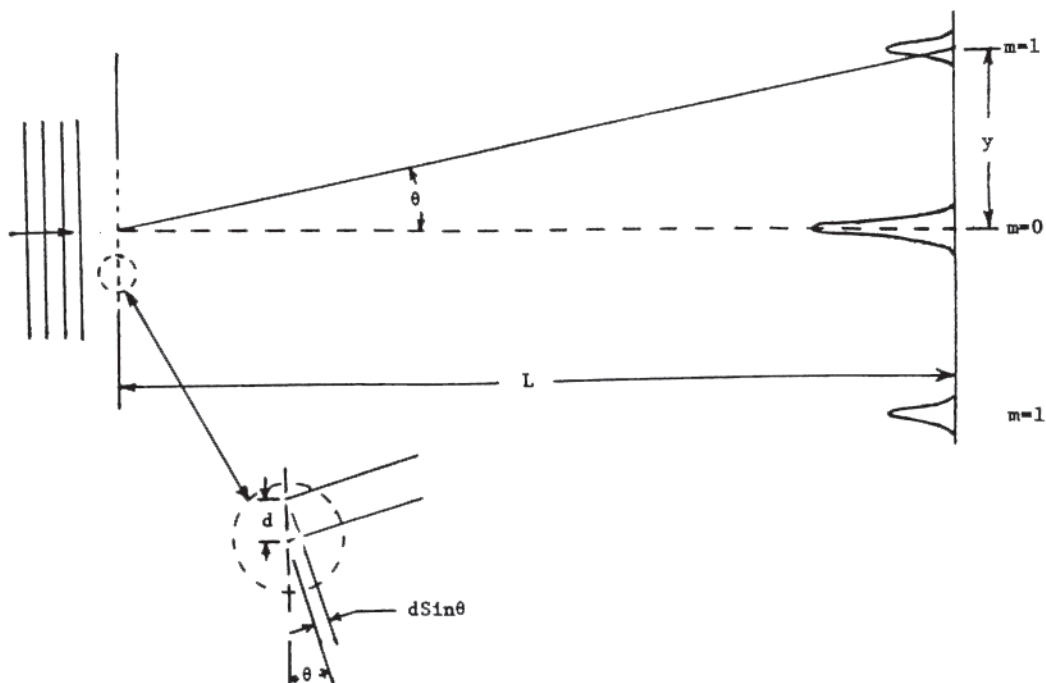

Figure 6

From the figure

(23) $$\text{Sin}\theta \;=\; \frac{y}{(y^2 + L^2)^{1/2}}$$

Combining Equations (**22**) and (**23**) gives

(24) $$\frac{y}{(y^2 + L^2)^{1/2}} = n\lambda N$$

or

(25) $$y \;=\; \frac{n\lambda L}{\left(\dfrac{1}{N^2} - n^2\lambda^2 \right)^{1/2}}$$

PROCEDURE

In this experiment we will use a Light Sensor to measure and record the diffraction pattern created by monochromatic laser light passing through an electroformed single-slit, double slit, and a diffraction grating. We will make a number of theoretical predictions and then compare our results with measurements taken from a printout of the diffraction patterns.

Part I. Computer and Interface Setup

1. Connect the PASCO Interface to the computer, turn on the interface, and turn on the computer.

2. Connect the Light Sensor DIN plug into Analog Channel A on the interface.

3. Connect the Rotary Motion Sensor stereo phone plugs to Digital Channels 1 and 2 (yellow plug in channel 1).

Part II. Equipment Setup

1. Mount the laser near the end of the optics bench.

2. Place the single-slit diffraction slide and component carrier so that the diffraction slide is at the 100 cm mark.

3. Turn on the **power switch** on the back of the laser. Rotate the position of the single-slit diffraction slide so that the laser beam passes through the slit marked a = 0.08 and forms a clear diffraction pattern. **Note:** the slit should be vertical and centered in the mounting bracket. A piece of white paper may serve as a temporary viewing screen to see the diffraction pattern.

4. Record the slit width "a" of the single-slit pattern you use in the Data section of your Lab Manual.

5. Position the **Light Sensor** and **Aperture Assembly** so that the aperture is at the 10 cm mark on the track. Turn the wheel of the **Rotary Motion Sensor** until you position the Light Sensor and Aperture at the midline of the track.

6. Rotate the **Aperture** disk mounted to the Light Sensor to the slit marked number 3.

7. Using the vertical and horizontal knobs on the back of the laser, adjust the laser so the diffraction pattern is centered on the Aperture slit number 3.

8. Turn the wheel of the **Rotary Motion Sensor** until the Light Sensor and Aperture Assembly stops at the left side of the mounting bracket.

9. **You do not need to calibrate the Light Sensor.** However, set the gain switch to 10. After your first data run, if the curve is not fairly smooth, you may need to adjust the gain to 100; if the peak is blocked off at the top, you may need to set the gain from 10 to 1. When in doubt, **ask your instructor** to look at your graph before printing.

Part III. Software Set-up

1. Start Data Studio, and select *Create Experiment*.

2. Inform the software which analog port you plugged the **Light Sensor** into by selecting the **Light Sensor** icon and dragging it to the appropriate analog port.

3. Inform the software which digital port you plugged the yellow wire of the **Rotary Motion Sensor** into by selecting the **Rotary Motion Sensor** icon and dragging it to the appropriate digital port. Note that the software will automatically indicate that the black wire is plugged into the adjacent digital port.

4. Double click on the **Rotary Motion Sensor** icon that is connected to the interface to open the *Sensor Properties* window.

5. Under the *General* tab, Change the **Sample Rate** to 100Hz. The radio button should already be selected as **Fast (>1Hz)**.

6. Under the **Measurement** tab, uncheck the *Angular Position Ch1&2 (deg)* box, and check the *Position Ch1&2 (m)* box.

7. Click **OK**.

8. Create a graph of Light Intensity.

9. Change the x-axis from time to position by dragging the **Position Ch1&2** icon from the data section to the x axis of the graph. When the x axis is enclosed in a dotted box, drop the icon.

10. Double click somewhere within the body of the actual graph, or click on the **Graph Settings** button located on the graph toolbar to open the Graph Settings window. Select the Tools tab. Under the Smart Tools, set the *Data Point Gravity* to 0. Select **OK.**

Part IV. Collecting Data for the Single Slit Pattern

1. With the **Rotary Motion Sensor** positioned at the left end of the mounting bracket, click the **Start** button.

2. Slowly slide the **Rotary Motion Sensor** to move the **Light Sensor** and aperture across the Diffraction pattern.

3. When the assembly stops at the right end, click the **Stop** button.

4. Click on the Graph window to make sure it is active, and maximize the window to fill the entire screen.

5. Press the autoscale button.

6. Use the Zoom Select option to magnify the central max, and the first and secondary maximums. (When repeating this process for the Double Slit Pattern, you should include the 3rd, 4th, and 5th secondary maximums in the magnification.)

7. If the graph is very jagged, you may need to change the gain of the **Light Sensor** to 100 and re-run the Diffraction pattern again. If the central max gets "blocked" off, you may need to adjust the gain of the"**Light Sensor** down and re-run the diffraction pattern again.

8. Print a copy of the graph for each member of your group.

Part V. Analyzing the Single Slit Data

1. With the graph window active, click on the **Smart Tool** button [icon]. When the smart tool curser has the following appearance [icon], it can be moved to any location on the graph.

2. Use the Smart Tool to determine the center of the Central Maximum, and manually record the position on your printed graph.

3. In addition, find the centers of each of the following, and record the values on you graph:
   a. Centers of <u>both</u> First Secondary Maximums
   b. Centers of <u>both</u> First Order Minimums (n = 1)
   c. Centers of <u>both</u> Second Order Minimums (n = 2)

   Note: When labeling the Double Slit Pattern, include through the 3rd Secondary maximums and 4th Minimums.

4. Complete the Single Slit portion of the Data and Calculation Sheet.

5. Before beginning the next slit pattern, the data from the previous slit pattern may need to be deleted from the graph, or a new graph can be created with the new slit pattern data.

Part VI. Recording the Double Slit Pattern

1. You will now repeat the above procedure using the double slit pattern with the same slit with of a=.08 and separation of d=.25.

2. Replace the Single Slit Disk and Bracket with Double Slit Disk and Bracket. Make sure the correct slit is positioned vertically in the center of the bracket as in Part A.

3. When magnifying the double slit pattern for analysis, be sure to include the region of the 3rd secondary maximums and 4th minimums.

4. Complete the Double Slit portion of the Data and Calculation Sheet.

Part VII. Recording the Diffraction Grating Pattern

1. Replace the Double Slit Disk and Bracket with the Bracket that has the Diffraction Grating taped to it.

2. Position the viewing screen (with a piece of white paper taped to it) in front of the Light Sensor and Aperture Assembly so it can be used to mark the locations of the Central Maximum and First Order Diffraction Pattern .

3. Mark these locations on the paper taped to the screen. Using a metric ruler, measure the distance from the central maximum to the First Order dots. Record this information as indicated in the data table in the lab manual.

4. Record the positions of the diffraction grating and screen. Determine the distance between them.

5. Complete the Diffraction Grating portion of the Data and Calculation sheet.

DATA AND CALCULATION SUMMARY

Distance from the single slit to the light sensor . . . . L_{SS} = _____

Distance from the double slit to the light sensor . . . . L_{DS} = _____

Distance from the grating to the viewing screen . . . . L_{G} = _____

Width of single slit and double slits . . . . a = _____

Separation between the two slits . . . . d = _____

Wavelength of laser light . . . . λ = _____

Number of lines on the diffraction grating . . . . N = _____

Theoretical Determination, Experimental Values, and a Comparison:

	Physical Quality	Equation used for the Theoretical Determination	Theoretical Value	Experimental Value	Percent Difference
Part I **Single Slit**	Location of the first order minima				
	Location of the second order minima				
	Location of the first secondary maximum				
Part II **Double slit**	Number of interference fringes under the central maximum				
	Interference fringe spacing				
Part III **Diffraction Grating**	Location of the first secondary maximum				